Handout 2

1 A Two-Country Model with Complete Markets

We are going to extend the closed economy RBC model described in the first handout and expanded in the homework to encompass trade in goods and assets across two countries. The main features of the model we are interested in setting up are the same as in the model by Baxter and Crucini (see reading list).

In our new model, each country specializes in the production of one good which is an imperfect substitute of the good produced abroad (this is one departure from Baxter and Crucini, who assume perfect substitutes). Both domestic and foreign goods are used for consumption. Only domestic goods are used for investment. We also assume that there is a complete set of state contingent assets that can be traded across countries. For now, the two countries are assumed to be an exact replica of each other. We'll relax this assumption later.

In each country there is a continuum of households of measure 1. Denote foreign variables with a "*" superscript. The maximization problem of the representative household in the Home country is given below:

$$
\begin{aligned}
&\max_{[C_t, I_t, K_{t+1}, B_{t+1}]} E_t \sum_{j=0}^{\infty} \beta^j \left(U(C_{t+j}) + V(L_{t+j}) \right) + \\
&+\beta^j \lambda_{t+j} \left[\Pi_t + w_{t+j} L_{t+j} + r_{kt+j} K_{t+j} + \right. \\
&-\frac{1}{2} \psi_K P_{q,t+j} K_{t+j} \left(\frac{I_{t+j}}{K_{t+j}} - \delta \right)^2 + \\
&\left. -P_{c,t+j} C_{t+j} - I_{t+j} - P_{Ft+j} \int_{s(t+j+1)} \psi^*_{t+j+1,t+j} B_{s(t+j+1)} + P_{Ft+j} B_{t+j} \right] \\
&+\beta^j \gamma_{t+j} \left[(1-\delta) K_{t+j} + I_{t+j} - K_{t+j+1} \right].
\end{aligned}
$$

Notice that there are complete markets because there are as many bonds as states of nature and because agents can trade bonds across countries. Bonds are denominated in units of the foreign good. P_{Ft} is the exchange rate, expressed as number of units of the foreign good per unit of the domestic good, while $\psi^*_{t+j+1,t+j}$ is the US dollar price of a bond and is, in turn, a function of the conditional probability that a certain state $s(t+j+1)$ will take place as will shall see below. A bond $B_{s(t+j+1)}$ pays one unit of the foreign good if state s occurs in period $t+j+1$, and 0 otherwise. I have dropped the state index from most of the notation used above, but every variable can be thought of as state dependent. The parameter psi_k affects the cost of adjustment for capital. The quadratic functional form for these costs conforms with the functional restriction assumed by Baxter and Crucini for this adjustment function. The representative household in the Foreign country faces exactly the same problem with the exception that the bonds are denominated in terms of the local good.

1.1 Bond Pricing, the Complete Market Condition, and the Risk Free Interest Rate

The FOC for consumption takes the form:

$$\frac{\partial}{\partial c_{t+j}} = U_{c,t+j} - \lambda_{t+j} P_{c,t+j} = 0, \tag{1}$$

Rearranging, $\lambda_{t+j} = \frac{U_{c,t+j}}{P_{c,t+j}}$. This is also true in the foreign country. Thus, $\lambda^*_{t+j} = \frac{U^*_{c,t+j}}{P^*_{c,t+j}}$. Reintroducing the dependence on the states of nature, the first order condition for a bond paying a unit of the foreign good in state $s(t+j+1)$, and 0 otherwise, is

$$\lambda_{s(t+j)}(h) P_{Fs(t+j)} \psi^*_{s(t+j+1),s(t+j)} = \beta Prob(s(t+j+1), s(t+j)) \lambda_{s(t+j+1)}(h) P_{Fs(t+j+1)}. \tag{2}$$

Similarly, for the foreign country,

$$\lambda^*_{s(t+j)}(h) \psi^*_{s(t+j+1),s(t+j)} = \beta Prob(s(t+j+1), s(t+j)) \lambda^*_{s(t+j+1)}(h). \tag{3}$$

N.B.: there is no exchange rate term here, because the bonds pay off in units of the foreign good. Combining the foreign first-order condition for consumption with the foreign first-order condition for bond holding yields:

$$\psi^*_{s(t+1),s(t)} = \beta \frac{\lambda^*_{s(t+1)}}{\lambda^*_{s(t)}} Prob(s(t+1), s(t)) = \tag{4}$$

$$\beta \frac{U^*_{c,t+1}}{U^* c, t} \frac{P^*_{c,t}}{P^*_{c,t+1}} Prob(s(t+1), s(t)). \tag{5}$$

The same condition in the home country looks like this:

$$\psi^*_{s(t+1),s(t)} = \beta\frac{\lambda_{s(t+1)}}{\lambda_{s(t)}}Prob(s(t+1),s(t)) = \tag{6}$$

$$\beta\frac{U^*_{c,t+1}}{U^*c,t}\frac{P_{Fs(t)}}{P_{Fs(t+1)}}\frac{P_{c,t}}{P_{c,t+1}}Prob(s(t+1),s(t)). \tag{7}$$

Define Q_t, the consumption-based real exchange rate, as $Q_t = \frac{P_{Ft}P^*_{c,t}}{P_{c,t}}$. Combining the bond pricing equations in the home and foreign country, one obtains:

$$\frac{\frac{U_{c,t+1}}{U_{c,t}}}{\frac{U^*_{c,t+1}}{U^*_{c,t}}} = \frac{\frac{e_t P^*_{c,t}}{P_{c,t}}}{\frac{e_{t+1}P^*_{c,t+1}}{P_{c,t+1}}} = \frac{Q_t}{Q_{t+1}}. \tag{8}$$

Given log utility,

$$\frac{\frac{c_t}{c_{t+1}}}{\frac{c^*_t}{c^*_{t+1}}} = \frac{Q_t}{Q_{t+1}}. \tag{9}$$

Iterating backwards yields

$$\frac{\frac{c_0}{c_{t+1}}}{\frac{c^*_0}{c^*_{t+1}}} = \frac{Q_0}{Q_{t+1}}, \tag{10}$$

which is equivalent to

$$\frac{c_{t+1}}{c^*_{t+1}} = Q_{t+1}\frac{c_0}{Q_0 c^*_0}, \tag{11}$$

and can be rewritten as:

$$\frac{c_t}{c^*_t} = \kappa Q_t, \tag{12}$$

where $\kappa = \frac{c_0}{Q_0 c^*_0}$.

1.2 Import and Export demand

Households take $P_{c,t}$ as given. We can think of an aggregator, with the same preferences as the representative agent, that solves:

$$\min_{C_{D,t},M_t} P_t C_{D,t} + P_{Ft}C_{Mt} + P_{Ct}\left[C_t - \left(\omega_c^{\frac{\rho_c}{1+\rho_c}}C_{D,t}^{\frac{1}{1+\rho_c}} + (1-\omega_c)^{\frac{\rho_c}{1+\rho_c}}C_{Mt}^{\frac{1}{1+\rho_c}}\right)^{1+\rho_c}\right]. \tag{13}$$

The the cost minimization problem yields the following conditions:

$$C_{D,t} = \omega_c \left(\frac{1}{P_{ct}}\right)^{-\frac{1+\rho_c}{\rho_c}} C_t \tag{14}$$

$$C_{Mt} = (1-\omega_c)\left(\frac{P_{Ft}}{P_{ct}}\right)^{-\frac{1+\rho_c}{\rho_c}} C_t \tag{15}$$

$$P_{ct} = \left[\omega_c + (1-\omega_c) P_{Ft}^{-\frac{1}{\rho_c}}\right]^{-\rho_c}. \tag{16}$$

1.3 Firms

In each country, there is also a continuum of firms of measure 1. In the The representative firm uses capital and labor to produce a final output good that can either be consumed or invested. In the Home country the representative firm has a production function of the form:

$$Y_t = f(K_t, L_t, M_t, N_t)$$
$$Y_t = \left[\nu \left(K_t e^{M_t}\right)^{1-\theta} + \left(L_t e^{N_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

where M_t and N_t are productivity processes given by:

$$M_{t+1} = \rho_M M_t + \epsilon_{Mt+1}$$
$$N_{t+1} = \rho_N N_t + \epsilon_{Nt+1}$$

where $|\rho_N| < 1$ and $|\rho_N| < 1$, and the innovations ϵ_{Mt} and ϵ_{Nt} are normally and independently distributed. The firm purchases capital services and labor in perfectly competitive factor markets, so that it takes as given the rental price of capital r_{kt} and the aggregate wage w_t. The firm chooses capital and labor so as to maximize profits $\Pi_t = Y_t - r_{kt}K_t - w_t L_t$, subject to the technology constraint.