Handout 3

1 Introducing nontraded goods in a two-country model

In both the home and the foreign country, there is a continuum of households of measure 1. The utility maximization problem for a representative household in the home country now takes the form

$$\max_{[C_t, I_t, K_{t+1}, B_{D,t+1}, B_{F,t+1}]} E_t \sum_{j=0}^{\infty} \beta^j \left(U(C_{t+j}) + V(L_{t+j}) \right) +$$
$$+\beta^j \lambda_{t+j} \left[\Pi_{t+j} + P_{Nt} w_{Nt+j} L_{Nt+j} + w_{Tt+j} L_{Tt+j} + P_{Nt} r_{Nt+j} K_{Nt+j} + r_{Tt+j} K_{Tt+j} \right.$$
$$-P_{Ct+j} C_{t+j} - P_{Jt+j} J_{Nt+j} - P_{Jt+j} J_{Tt+j} - \frac{P_{Ft+j}}{(1+i_t*)\psi(B_{Ft+j})} B_{Ft+j+1} + P_{Ft+j} B_{Ft+j}$$
$$\left. - \int_{s(t+j+1)} \psi_{t+j+1,t+j} B_{Dt+j+1} + B_{D,t+j} \right] +$$
$$+\beta^j \gamma_{Nt+j} \left[(1-\delta) K_{Nt+j} + J_{Nt+j} - \psi_k K_{Nt+j} \left(\frac{J_{Nt+j}}{K_{Nt+j}} - \delta \right)^2 - K_{Nt+j+1} \right]$$
$$+\beta^j \gamma_{Tt+j} \left[(1-\delta) K_{Tt+j} + J_{Tt+j} - \psi_k K_{Tt+j} \left(\frac{J_{Tt+j}}{K_{Tt+j}} - \delta \right)^2 - K_{Tt+j+1} \right]$$
$$+\beta^j \eta_{t+j} \left[L_{t+j} - L_{Nt+j} - L_{Tt+j} \right].$$

In the maximization problem above, a "T" subscript denotes variables for the sector producing traded goods, and a "N" subscript denotes variables for the nontraded sector. We are now keeping track of two separate capital stocks. In period t, the capital stock is predetermined, hence it cannot be reallocated across sectors. However, in subsequent periods, because we are not imposing constraints on investment, our setup does not rule out capital mobility across sectors. The costs of adjustment for capital, parameterized through ψ_k, is a way to slow down capital reallocation across sectors. Investment is denoted by the variable J. By contrast, labor is free to move across sectors.

The budget constraint is expressed in units of good produced by the home traded sector. Investment is now denoted by the variable J. Like consumption it is a composite of inputs purchased from the nontraded sector, the domestic traded sector and the foreign traded sector. More details on this are below.

As in the setup for the model described in Handout 3, there are complete financial markets within a country. However, across countries agents only trade a risk-free bond. The function ψ makes the interest rate debt-elastic, so as to ensure stationarity for the equilibrium holding of the foreign risk-free bond. Our representative agent setup implies that the only bond to be traded in equilibrium is the foreign risk-free bond.

The foreign problem is analogous.

1.1 Households' cost minimization problem

Consider the consumption good first.

We are going to assume that consumption is a composite of nontraded goods and traded good component. In turn, the traded good component is a composite of the domestic traded good and the foreign traded good. At each stage, we'll use a CES function to aggregate the components. This allows us to differentiate between the the degree of substitutability for the home and the foreign goods, and the degree of substitutability for the traded and the nontraded goods.

Consider the tradeable component first. The cost minimization problem is the following:

$$\min C_{THt} + P_{Ft}C_{TMt} + P_{Tt}\left[C_{Tt} - \left(\omega_{CT}^{\frac{\rho_T}{1+\rho_T}} C_{THt}^{\frac{1}{1+\rho_T}} + (1-\omega_{CT})^{\frac{\rho_T}{1+\rho_T}} C_{TMt}^{\frac{1}{1+\rho_T}}\right)^{1+\rho_T}\right]. \tag{1}$$

Where C_{THt} denotes the home-produced traded good, C_{TMt} the foreign traded good, C_{Tt} is the traded good bundle, and P_{Tt} can be interpreted as the price of the traded bundle. As before, P_{Ft} is the domestic price of the foreign traded good. The first-order conditions for this problem imply:

$$C_{THt} = \omega_{CT}\left(\frac{1}{P_{Tt}}\right)^{-\frac{1+\rho_T}{\rho_T}} C_{Tt} \tag{2}$$

$$C_{TMt} = (1-\omega_{CT})\left(\frac{P_{Ft}}{P_{Tt}}\right)^{-\frac{1+\rho_T}{\rho_T}} C_{Tt} \tag{3}$$

$$P_{Tt} = \left[\omega_{CT} + (1-\omega_{CT}) P_{Ft}^{-\frac{1}{\rho_T}}\right]^{-\rho_T}. \tag{4}$$

Consider the final consumption bundle next. The cost minimization problem is the following:

$$\min P_{Nt}C_{Nt} + P_{Tt}C_{Tt} + P_{Ct}\left[C_t - \left(\omega_{CN}^{\frac{\rho_N}{1+\rho_N}} C_{Nt}^{\frac{1}{1+\rho_N}} + (1-\omega_{CN})^{\frac{\rho_N}{1+\rho_N}} C_{Tt}^{\frac{1}{1+\rho_N}}\right)^{1+\rho_N}\right]. \tag{5}$$

$$C_{Nt} = \omega_{CN}\left(\frac{P_{Nt}}{P_{Ct}}\right)^{-\frac{1+\rho_N}{\rho_N}} C_t \tag{6}$$

$$C_{TMt} = (1-\omega_{CN})\left(\frac{P_{Ft}}{P_{Ct}}\right)^{-\frac{1+\rho_N}{\rho_N}} C_t \tag{7}$$

$$P_{Ct} = \left[\omega_{CN} P_{Nt}^{-\frac{1}{\rho_N}} + (1-\omega_{CT}) P_{Tt}^{-\frac{1}{\rho_N}}\right]^{-\rho_N}. \tag{8}$$

The cost minimization problem for investment is analogous, but allows for investment specific ω parameters determining the intensity of the various inputs.

1.2 The firms' problem

In both sectors, there is a continuum of firms of measure 1. Firms are price takers in both the input and the product markets. In the traded sector, a representative firm's cost minimization problem is the following:

$$\min r_{Tt}K_{Tt} + w_{Tt}L_{Tt} + \mu_t\left[Y_{Tt} - K_{Tt}^{\alpha}L_{Tt}^{1-\alpha}\right] \tag{9}$$

The first-order conditions from the above problem imply that:

$$r_{Tt} = \alpha\frac{Y_{Tt}}{K_{Tt}}$$

$$w_{Tt} = (1-\alpha)\frac{Y_{Tt}}{L_{Tt}}$$

Firms in the nontraded sector face an analogous cost minimization problem.

1.3 Resource constraints, net foreign assets, and the UIP condition

The resource constraints on sectors N and T are such that:

$$Y_{Tt} = C_{THt} + I_{THt} + C^*_{TMt} + I^*_{TMt} \tag{10}$$

$$Y_{Nt} = C_{TNt} + I_{TNt}. \tag{11}$$

Notice that given that we have two sectors, there is no unique way of defining aggregate output. One way, is to express aggregate output in units of traded goods:

$$Y_t = Y_{Tt} + P_{Nt}Y_{Nt}.$$

The net-foreign asset condition can be found by aggregating over all households' budget constraints, so that:

$$P_{Ft}\frac{B_{Ft+1}}{(1+i^*_t)\psi(B_{Ft})} = P_{Ft}B_{F,t} + C^*_{TMt} + I^*_{TMt} - P_{Ft}C_{TMt} - P_{Ft}I_{TMt} \tag{12}$$

1.4 Linearizing the consumption-based real exchange rate

Let the consumption-based real exchange rate be defined as $Q_t = P_{Ft}\frac{P_{Ct^*}}{P_{Ct}}$. Let a $\hat{}$ over a variable denote that variable's deviation from its steady state. In a steady state such that all relative prices are unitary,

$$\hat{Q}_t = \hat{P}_{Ft} + \hat{P}^*_{Ct} - \hat{P}_{Ct}. \tag{13}$$

From the linearization of equation (8), we can obtain:

$$\hat{P}_{Ct} = \omega_{CN}\hat{P}_{Nt} + (1-\omega_{CN})\hat{P}_{Tt}. \tag{14}$$

Similarly, from equation (4)

$$\hat{P}_{Tt} = (1-\omega_{CT})\hat{P}_{Ft}. \tag{15}$$

Combining the two equations immediately above,

$$\hat{P}_{Ct} = \omega_{CN}\hat{P}_{Nt} + (1-\omega_{CN})(1-\omega_{CT})\hat{P}_{Ft}. \tag{16}$$

Similarly, for the foreign country:

$$\hat{P}^{*}_{Ct} = \omega_{CN}\hat{P}^{*}_{Nt} + (1-\omega_{CN})(1-\omega_{CT})\hat{P}^{*}_{Ft}. \tag{17}$$

But the equation above, can be rewritten as:

$$\hat{P}^{*}_{Ct} = \omega_{CN}\hat{P}^{*}_{Nt} - (1-\omega_{CN})(1-\omega_{CT})\hat{P}_{Ft}. \tag{18}$$

Combining (13) with (17) and (18), we can see that:

$$\hat{Q}_{t} = \omega_{CN}(\hat{P}^{*}_{Nt} - \hat{P}_{Nt}) + (1-(1-\omega_{CN})(1-\omega_{CT}))\,\hat{P}_{Ft} \tag{19}$$

The introduction of of nontraded goods has broken the correlation between the production-based real exchange rate and the consumption-based real exchange rate. When we introduce departures from the law of one price, we'll have a further channel to break this correlation.

Using the dynare programs when nontraded goods account for about 50% of the economy, compare the effects of a technology shock in the traded sector when nontraded goods are complements, and when they are substitutes for traded goods. Also, consider the effects of reducing the size of the nontraded sector.