Econ 642, Spring 2007.

Homework 1

Question 1

Consider the following model.

Households

There is a continuum of households of measure 1. The utility function of the representative household is

$$U = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\log(C_s) + \chi_0 \frac{(1-L_s)^{1-\chi} - 1}{1-\chi} \right] \tag{1}$$

where the discount factor β satisfies $0 < \beta < 1$ and E_t is the expectation operator conditional on information available at time t. The period utility function depends on consumption, C_s, and labor, L_s.

Households can rent labor and capital services to firms in competitive markets. The representative household earns labor income of $w_t L_t$, and a rental rate on capital of r_t. It also receives an aliquot share of firms' profits Π_t.

The representative household's budget constraint in period t is such that its expenditure on consumption and investment goods (I_t) must equal its disposable income. That is,

$$r_t K_t + w_t L_t + \Pi_t = C_t + I_t. \tag{2}$$

Purchases of investment goods augment the household's capital stock according to the accumulation equation:

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{3}$$

where δ is the depreciation rate.

In every period t, the representative household maximizes utility (1) with respect to its consumption, labor supply, investment, (end-of-of-period) capital stock, subject to its budget constraint (2) and the transition equation for capital (3).

Firms

There is a continuum of firms of measure 1. The representative firm uses capital and labor to produce a final output good that can either be consumed or invested. This firm has a production function of the form:

$$Y_t = f(K_t, L_t, M_t, N_t)$$
$$Y_t = \left[\nu\left(K_t e^{M_t}\right)^{1-\theta} + \left(L_t e^{N_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

where M_t and N_t are productivity processes given by:

$$\begin{aligned} M_{t+1} &= \rho_M M_t + \epsilon_{Mt+1} \\ N_{t+1} &= \rho_N N_t + \epsilon_{Nt+1} \end{aligned}$$

where $|\rho_N| < 1$ and $|\rho_N| < 1$, and the innovations ϵ_{Mt} and ϵ_{Nt} are normally and independently distributed. The firm purchases capital services and labor in perfectly competitive factor markets, so that it takes as given the rental price of capital r_t and the aggregate wage w_t. The firm chooses capital and labor so as to maximize profits $\Pi_t = Y_t - r_t K_t - w_t L_t$, subject to the technology constraint.

Question 1.1

Find the first-order conditions for the representative household's utility maximization problem. Hint: notice that $\Pi_t = 0$ in equilibrium.

Question 1.2

Find the first-order conditions for the representative firm's profit maximization problem

Question 1.3

List the necessary conditions for the model's equilibrium.

Question 1.4

Let χ_0 be a free parameter that you can choose to ensure that the steady state level of labor supply is some desired value L^*. Find the steady state values for all the variables in the model as a function of the model's parameters and L^*.

Question 1.5

Solve for χ_0 in terms of L^* and of other parameter values.

Question 1.6

Let ν be a free parameter that you can choose to ensure that the steady state share of capital income ($r^* \frac{K^*}{Y^*}$) equals some desired level S_K. Solve for ν in terms of other parameters and S_K only.

Question 1.7

Linearize the necessary conditions for an equilibrium. Show that the choice of χ_0 cannot influence the model dynamics, other than through its effect on L^*.

Question 1.8

Let $\delta = 0.025$, $\beta = 0.99$, $\chi = 10$, $\theta = 2$.

Write a matlab program that finds a solution to the system of linear difference equations characterizing the model's equilibrium using the generalized complex Schur decomposition we studied in class.

Question 1.9

Compare the reaction of the economy to labor and capital augmenting technology shocks that raise output by 1 percent (relative to steady state) on impact.

Question 2

Consider the same model you analyzed in question 1, but let $\rho_N = 0.5$. Using Dynare, assuming perfect foresight, obtain the nonlinear response of the economy to a labor-augmenting technology shock that raises output by 1% on impact. Compare this response to what you would obtain letting Dynare linearize the model. Plot the responses of output, consumption, investment, capital, and labor that obtained through the two solution procedures.

Question 3

Verify graphically that as the size of the shock increases the linear approximation deteriorates (e.g., repeat the exercise in question 2 for shocks that raise output by 50% on impact).